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A Numerical Model of Externally Prestressed Concrete Beam

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Abstract— In an external prestressing system, there is no strain compatibility between the cable and the concrete at every cross-section, the increment of cable strain must be evaluated by taking into account the whole structure, rather than performing the calculation at each section, independently. In this study, a method for the calculation of cable strain, which is based on the deformation compatibility of beam and friction at the deviators, was proposed to predict entire response of externally prestressed concrete beams up to elastic limit. Application of the developed method in numerical analysis on a rectangular beam with different profiles of prestressing cable was then performed. An algorithm has been developed to determine the structural behaviour at the deviator points in an externally pre-stressed beam. The predicted results showed that the structural behavior of externally prestressed concrete beams could be satisfactorily predicted from zero loading stage up to the proportional limit loading stage for different cable profiles.The results obtained have been verified by subsequent modeling of the externally pre-stressed beam in ABAQUS 6.10 CAE platform. A comparative study has been carried out to justify the most effective cable profile for most effective prestressing.

Index Terms— Cable Strain, Compatibility, Deformation, Deviator, Elastic Limit, External Cable, External Prestressing.

1 INTRODUCTION

EXTERNAL prestressing is defined as prestress introduced by the high strength cable which is placed outside the cross section and attached to the beam at some deviator points along the beam. The use of external prestressing is gaining popularity in bridge constructions because of its simplicity and cost-effectiveness. Moreover, the external prestressing is applied not only to new structures but also to existing structures which need to be repaired or strengthened. Although various advantages of external prestressing have been reported elsewhere, Some questions concerning the behavior of externally prestressed concrete beams are often arisen in the design practice.

One of major problems concerning the beams prestressed with external cables is in calculating the cable stress beyond the effective prestress. In the case of beams prestressed with bonded cables, since the cable strain is assumed to be the same as the concrete strain at the cable level, the calculation of cable strain under the applied load is a problem related only to a section of maximum moment, i.e., the increase of cable strain is section-dependent. This is totally different in the case of beams prestressed with external cables. Since the cable is unbonded, the cable freely moves in the relation of beam deformation. Therefore, the cable strain is basically different from the concrete strain at every cross section, i.e., the cable strain cannot be determined from the local strain compatibility between the concrete and the cable. For the calculation of cable strain, it is necessary to formulate the global deformation compatibility of beam between the extreme ends. The strain variation in an external cable should be considered to be a function of the overall deformation of beam. This means that the strain change in the cable is member-dependent, and is influenced by the initial cable profile, span to depth ratio, deflected shape of the structure, friction at the deviators, the initial condition of beam, etc. This makes the analysis of a beam with external cables more complicated, and proper modelling of the overall deformation of beam becomes necessary.

Since the prestressing force transfers to the concrete beam through the deviator points and anchorage ends, the cable friction obviously exists at the deviators, resulting in a different level of strain increase between the two successive cable segments. The increment of cable strain can be expressed as:

$$\Delta\varepsilon_s = \frac{1}{l}\int_0^l \Delta\varepsilon_{cs}\,dx \qquad (1)$$

Where $\Delta\varepsilon_s$ and $\Delta\varepsilon_{cs}$ are the increments of cable strain and concrete strain at the cable level, respectively; l is the total length of cable between the extreme ends.

If the cable is considered perfectly fixed at the deviators, meaning that the strain variation for each segment is independent from the others. The increment of cable strain depends only on the deformations of two successive deviators or anchorages, at which the cable is attached. The strain variation can be expressed as:

$$\Delta\varepsilon_{si} = \frac{\Delta l_i}{l_i} \qquad (2)$$

The strain compatibility principle considering the frictional force has been implemented for three different types of cable profiles such as parabolic, trapezoidal and straight. The effectiveness of those cable profiles is analysed in terms of ultimate mid-span deflection.

2 METHOD OF ANALYSIS

To obtain a whole deformed shape of beam, a finite element method is commonly used as one of powerful and popular tools in the structural analysis. The conventional finite element method often approximates a deformed shape of beam element with interpolation functions such as a cubic polynomial function for transverse displacement and a linear function for longitudinal displacement. The cubic function implies a linear variation of curvature along the element. However, the analysis of unbonded beams in general or the analysis of externally prestressed concrete beams in particular necessitates an

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accurate evaluation of strain variation in the concrete since the compatibility equation should be formulated with the values of concrete strain at the level of cable. Thus, a large number of short elements are necessary for the adequate evaluation of cable strain.

The program is capable of accounting for the flexural deformation, friction at the deviators, and external cables with different configuration (parabolic, trapezoidal, straight profile). In the analysis, the beam was represented by a set of beam elements connected together by nodes located at either end. Each node has three degrees of freedom, namely, horizontal displacement, vertical displacement and rotation. Incemental cable strain has been calculated considering the strain compatibility between the cable and the concrete beam at the deviator points within the elastic limit and the incremental stress in cable has been estimated thereafter. The estimated cable stress has been applied on the prestessing tendons in the analysis.Cross section of the beam was divided into layers, in which each layer might have different materials, but its properties were assumed to be constant over the layer thickness. The concrete strain of each layer for every beam element was determined, and appeared to take as the initial condition of beam. Thereafter the cable strain is calculated assuming free slip condition considering the frictional force. In this study, the only one displacement control at the mid span of the beam, was applied in the analysis.

2.1 Strain variation in external cables

2.1.1 Force equilibrium at a deviator

Figure 1 showed that *Fi*, *Fi*+1 are tensile forces in the cable segments (i) and (i+1) at the deviator (i). Correspondingly, θ*i*, θ*i*+1 are cable angles, respectively. Thus, the force equilibrium condition on the X direction can be expressed as:

$$F_i\cos\theta_i+(-1)^{k_i}\mu\left(F_i\sin\theta_i+F_{i+1}\sin\theta_{i+1}\right)=F_{i+1}\cos\theta_{i+1} \qquad (3)$$

where coefficient *ki* depends on the slipping direction, and has value *ki*=1 if *Fi*cosθ*i* > *Fi*+1cosθ*i*+1 and *ki*=2 if *Fi*cosθ*i* < *Fi*+1cosθ*i*+1; μ is the friction coefficient at the deviator.

Eq.(3) can be rewritten in terms of incremental forces as:

$$\Delta F_i\cos\theta_i+(-1)^{k_i}\mu\left(\Delta F_i\sin\theta_i+\Delta F_{i+1}\sin\theta_{i+1}\right)=\Delta F_{i+1}\cos\theta_{i+1} \qquad (4)$$

where Δ*Fi*, Δ*Fi*+1 are the incremental forces at the both sides of deviator.

Since the stress of an external cable usually remains below the elastic limit up to the failure of the beam, it is possible to rewrite the force equilibrium condition at the deviator in terms of the increments of cable strain by dividing both sides of Eq.(4) by E_sA_s, the force equilibrium condition can be then expressed as:

$$\Delta\varepsilon_{si}\cos\theta_i+(-1)^{k_i}\mu\left(\Delta\varepsilon_{si}\sin\theta_i+\Delta\varepsilon_{si+1}\sin\theta_{i+1}\right)=\Delta\varepsilon_{si+1}\cos\theta_{i+1}$$

or

$$\left[\cos\theta_i+(-1)^{k_i}\mu\sin\theta_i\right]\Delta\varepsilon_{si}+\left[-\cos\theta_{i+1}+(-1)^{k_i}\mu\sin\theta_{i+1}\right]\Delta\varepsilon_{si+1}=0 \qquad (5)$$

Where *Eps* and *Aps* are the elastic modulus and area of the cable; $\Delta\varepsilon s_i$, $\Delta\varepsilon s_{i+1}$ are the increments of cable strain at the both sides of deviator, respectively.

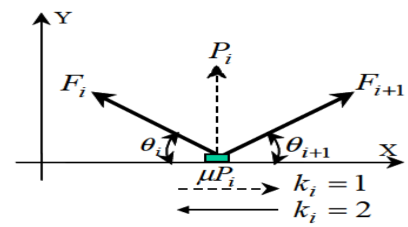

Fig.1 - Force equilibrium at a deviator

2.1.3 Proposed equation for cable strain

Since the deflection of external cable does not follow the beam deflection except at the deviator points during the beam is being deformed, the strain in a cable totally differs the strain in the concrete at the cable level. The strain induced in the concrete at the cable level varies according to the bending moment diagram, while the strain in an external cable is uniform over the length of cable segment between two successive deviators or anchorage end. The cable strain, therefore, cannot be determined from the local compatibility of deformation. An analytical model for externally prestressed concrete beams, therefore, should satisfy the total compatibility requirement, i.e., the total elongation of a cable must be equal to the integrated value of concrete deformation at the cable level. This is referred to as "deformation compatibility of beam" in this study. The mathematical expression of the deformation compatibility of beam is expressed as:

$$\sum_{i=1}^{n} l_i\,\Delta\varepsilon_{si} = \int_0^l \Delta\varepsilon_{cs}\,dx \qquad (6)$$

where Δε*si* is the increment of cable strain; *li* is the length of cable segment under consideration; Δε*cs* is the increment of concrete strain at the cable level. Combining Eq.(6) with the force equilibrium condition at the deviator, which is expressed in Eq.(5), one can analytically obtain the increment of cable strain of each segment at the certain loading stage, and it can be expressed as the following:

$$\begin{bmatrix} l_1 & l_2 & l_3 & \dots\dots\dots & l_{n-1} & l_n \\ C_1+(-1)^{k_1}\mu S_1 & -C_2+(-1)^{k_1}\mu S_2 & 0 & \dots\dots\dots & 0 & 0 \\ 0 & C_2+(-1)^{k_2}\mu & & & & 0 \\ \dots & \dots & & & & \dots \\ \dots & \dots & \dots & \dots\dots\dots & \dots & \dots \\ 0 & 0 & 0 & \dots\dots\dots & -C_{n-1}+(-1)^{k_{n-2}}\mu S_{n-1} & 0 \\ 0 & 0 & 0 & \dots\dots\dots & C_{n-1}+(-1)^{k_{n-1}}\mu S_{n-1} & -C_n+(-1)^{k_{n-1}}\mu S_n \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\varepsilon_3 \\ \dots \\ \dots \\ \Delta\varepsilon_{n-1} \\ \Delta\varepsilon_n \end{Bmatrix} = \begin{Bmatrix} \int \Delta\varepsilon_{cs}dx \\ 0 \\ 0 \\ \dots \\ \dots \\ 0 \\ 0 \end{Bmatrix}$$

Or

$$[M]\{\Delta\varepsilon_s\}=[N]\{d\}$$

$$\{\Delta\varepsilon_s\}=[M]^{-1}[N]\{d\}=[C]\{d\} \qquad (8)$$

where the letters Ci and Si are denoted as cosine and sine of the cable angle, and the subscripts under these letters indicate the cable angle number; {d} is the increment of nodal displacement vector.It can be seen from Eq.(7) that the strain variation in an external cable depends mainly on the overall deformation of the beam, friction at the deviators and cable angle. The increasing beam deformation under the applied load is in the relative change of cable elongation. The adequate evaluation of cable strain depends on the accurate extent in the calculation of concrete strain at the cable level. That is the strain variation in a cable depends on the displacement of every point of beam. Therefore, the concrete beam should be necessarily divided into a large number of short elements by using the finite element method.

3. NUMERICAL MODEL STUDY

A uniformly distributed load of 22 KN/m is applied on the entire span of the beam. Two noded linear beam element is used with appropriate meshing compatible with deviator locations of the beam. The evaluated pre-stressing force has been applied at those deviator points as equivalent nodal forces resolving it into two componenets to find the effectiveness of the calculated prestressing force with different cable profiles.

TABLE-1

DESCRIPTION OF THE PROPOSED MODEL

Property	Details
Beam Section	250X400 mm
Length	6 Meters
Material	Concrete
Density	2500 kg/m3
Poission's Ration	0.2
Support Condition	Simply supported
Imposed Load(UDL)	22 KN/m

A total of three externally prestressed concrete beams with different cable profile (parabolic, trapezoidal, straight) and same loading conditions, were considered to analyze as numerical examples in this study. They are simply supported beams. The compressive strength of concrete for these beams is 35.0 N/mm2. The material properties of the beams are shown in table1.

As mentioned earlier, there is a friction between the cables and the deviators, and the friction can be expressed in terms of the friction coefficient, μ as shown in Eq.(3). The real value of the friction coefficients depends on many factors, and it can only be determined by experimental investigations. However, the friction coefficients were not easy to find in any of the available literature. For analytical purposes, the friction coefficients at the deviators were assumed to have a certain value, and they were about 0.2, .25, .3 for the beamswith different cable profiles as mentioned. These values may be not true in actual; they were, however, only adopted for the purpose of numerical analysis.

4. GENERAL DISCUSSIONS OF ANALYTICAL RESULTS

An algorithm (based on finite element method) has been developed to calculate the incremental strain at the level of deviators points.
It is also capable to evaluate the incremental cable strain due to the applied loading condition. Incremental cable stress is calculated thereafter. Fig.2 to Fig.4 represent the variation of cable strain with various frictional coefficient (μ=.2, μ=.25, μ=.3) respectively at different deviator points.

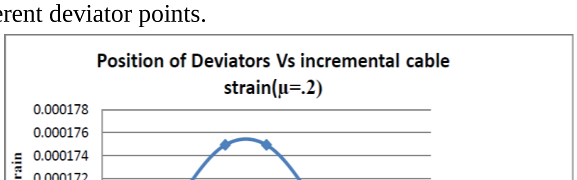

Fig.2 Variation of incremental cable strain throughout the span for various cable profiles (μ=.2)

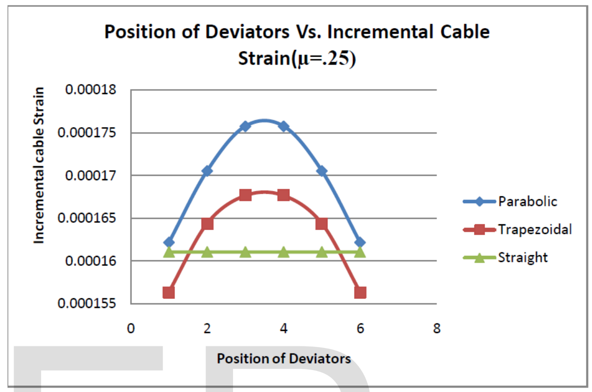

Fig.3 Variation of incremental cable strain throughout the span for various cable profiles (μ=.25)

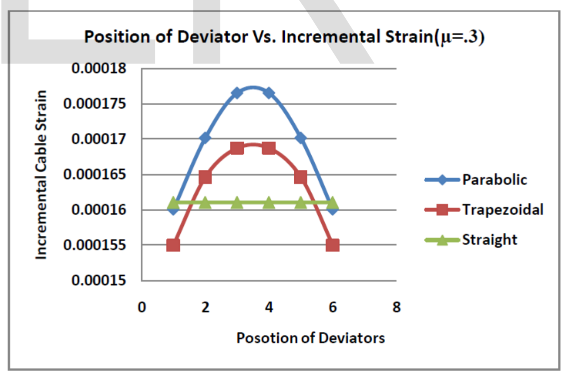

Fig.4 Variation of incremental cable strain throughout the span for various cable profiles (μ=.3)

The maximum values of incremental cable stresses at different anchorage locatios with different frictional coefficients are tabulated in table 2.

TABLE-2

Maximum Incremental Stresse for Different Cable Profiles with Different Frictional Coefficient.

Frictional coefficient(μ)	Max. Stress in Different Cable Profile(in Mpa)		
	Parabolic	Trapezoidal	Straight
.2	35.264	33.315	32.210
.25	35.595	33.531	32.210
.3	35.745	33.746	32.210

The obtained incremental cable stress would be induced in the pre-

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stressing cable due to the applied load. This stress is applied in the cable in opposite direction of the induced stress due to imposed load. The performance of the prestressed beams with various cable profiles is judged by evaluationg the final midspan deflection of the beam.
The beams with different cable profiles are subsequently modeled in ABAQUS 6.10 CAE platform. A uniformly distributed load of 22 KN/m is applied as uniformly distributed load. Eight noded brick element is used with appropriate meshing compatible with deviator locations of the beam. The evaluated pre-stressing force has been applied at those deviator points as equivalent nodal forces.

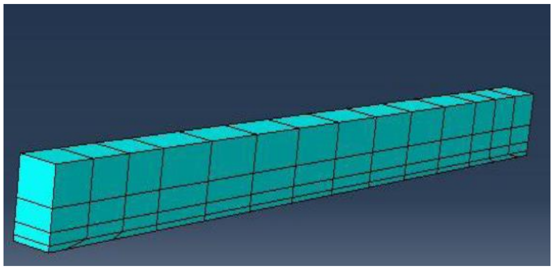

Fig.5 Finite element model of the beam with meshin in ABAQUS

The final midspan deflection obtained from the analysis in ABABQUS, have been compared with the final deflection obtained from the proposed algorithm. The predicted values of mid-span deflection are found in tune with the ABAQUS results.

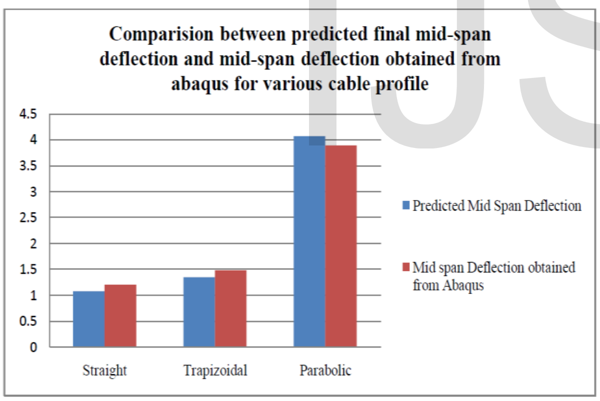

Fig.6 Comparision between predicted final mid-span upliftment and mid-span upliftment obtained from abaqus for various cable profiles.

5. CONCLUSIONS

Using a finite element algorithm the deflection of the anchorage point due to applied load is evaluated. The strain variation in an external cable was investigated on the basis of the deformation compatibility of beam. The following conclusions can be made in this study:
1. The proposed method for the numerical analysis can satisfactorily predict the behavior of externally prestressed concrete beams up to propotional limit. The predicted prestressing forces for various cable profiles are capable to bring back the deflected beam in its intial position
2. The stress increase in an external cable depends mainly on the overall deformation of beam and cable friction at the deviators.
3. As the maximum upliftment can be found for the parabolic profile of the cable, it can be concluded that the parabolic profile is most efficient among all the cable profiles discussed for the concerened beam.
4. Friction at the deviator points plays no role in case of straight cable profile whereas minute changesin incremental cable stress can be observed for parabolic and trapezoidal profiles.
5. The proposed method is generally suitable for the investigation of all kinds of beam prestressed with external cables with different types of cable profiles in simply supported or multispan continuous beams.
6. The final deflection, increase in cable starin and incremental cable stress for externally prestressed beam can be efficiently evaluated with the help of developed algorithm.

REFERENCES

[1]Aravinthan, T., Mutsuyoshi, H., Fujoka, A., Hishiki, Y., ”Flexural behavior of two span continuous segmental prestressed concrete beams with external tendons”, Proceeding of *JCI, 1996, Vol.18, No.2, pp.1121-1126*.

[2] Aravinthan, T., Mutsuyoshi, H., Niitsu, T., Chen, An., ”Flexural behavior of externally prestressed concrete beams with large eccentricities”, *Proceeding of JCI, 1998, Vol.20, No.3, pp.673-678.*

[3] Diep, B.K, “Non Linear Analysis of externally prestressed concrete continuous beams considering shear deformation”, *Master Thesis of Department of Civil Engineering-Nagoya University, 2000.*

[4] Diep, B.K., Tanabe, T., “Analysis of two continuous span prestressed concrete beam with external cables considering shear deformation”, *Proceeding of JCI, 1999, Vol.21, No.3, pp.955-960.*

[5] Garcia-Vargas, J.A., Menezes, N, Trinh, J.L., “Effect of external tendon slipping at deviator on beam behaviour”, *Proceeding of the Workshop on Behavior of External Prestressing in Structures, France, 1993, pp.227-237.*

[6] Lee, L.H., Moon, J.H., Lim, J.H., “Proposed methodology for computing of unbonded tendon stress at flexural failure”, *ACI Structural Journal, Nov-Dec, 1999, pp 1040-1048.*

[7] Macgregor,.R.J.G., Kreger M.E., Breen J.E., “Strength and ductility of three span externally post tensioned segmental box girder bridges model”, *Proceeding of External Prestressing in Bridges, 1990, Sp120-15, pp.315-338.*

[8] Harajli, M.H., “Effect of span-depth ratio on the ultimate steel stress in unbonded prestressed concrete members”, *ACI Structural Journal, May-June, 1990, pp.305-312.*

[9] Mutsuyoshi, H., Tsuchida, K., Matupayont, S., Machida, A., “Flexural behavior and proposal of design equation for flexural strength of externally prestressed concrete members”, *JSCE Journal of Materials, Concrete Structures and Pavements, 1995, Vol.26, No.508, pp.67-77.*

[10] Paulo Chaves de Rezende-Martins, Paulo de Araujo Regis, Jean-Marie Desir, “A study on the behavior of hyperstatic concrete beams with mixed prestressing”, *Proceeding of the Workshop on Behavior of External Prestressing in Structures, France, 1993, pp.217-226.*